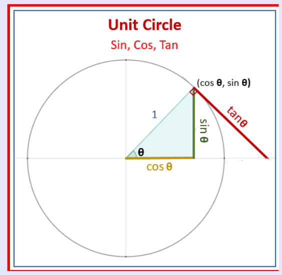
Trigonometry

Lecture 43



Feb 19-8:47 AM

Intro to Vectors:

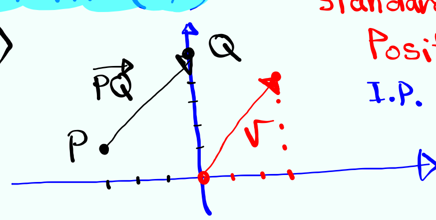
Vector is a directed line Segment with initial point $P(x_1, y_1)$ and terminal point $Q(x_2, y_2)$.

$$\overrightarrow{PQ} = \langle x_2 - x_1 , y_2 - y_1 \rangle$$

ex: Draw the vector with initial point $(-3, 1)$ and terminal Point $(0, 5)$

$\overrightarrow{PQ} = \langle 0-(-3), 5-1 \rangle$

$= \langle 3, 4 \rangle$



standard Position

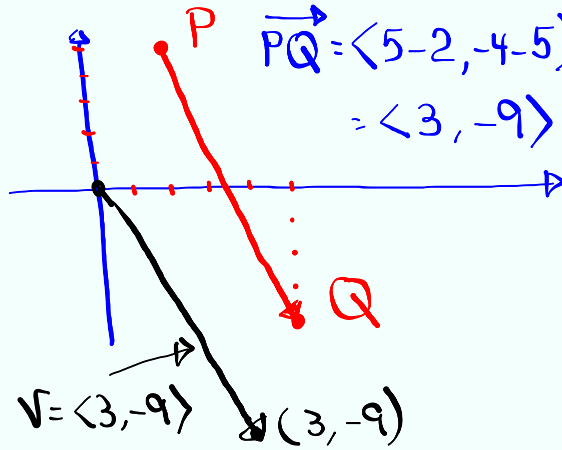
I.P. $(0,0)$

Nov 18-10:29 AM

Draw the vector $\overrightarrow{PQ}$ with $P(2,5)$ and $Q(5,-4)$.

Draw the vector in standard position as well.

$\overrightarrow{PQ} = \langle 5-2, -4-5 \rangle$
$= \langle 3, -9 \rangle$



$\vec{v} = \langle 3, -9 \rangle$

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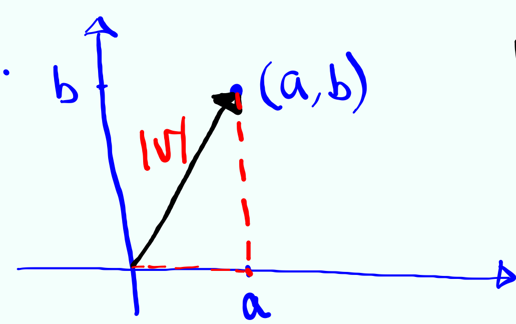
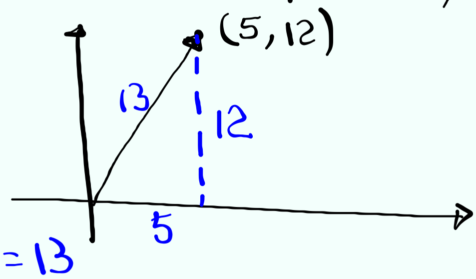
Draw vector $v = \langle a, b \rangle$ in standard position in QI.



Magnitude of V is $|\vec{v}| = \sqrt{a^2+b^2}$

Draw $V = \langle 5, 12 \rangle$ in standard position, find $|\vec{v}|$



$|\vec{v}| = \sqrt{a^2+b^2} = \sqrt{5^2+12^2}$
$= \sqrt{169} = 13$

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$u = \langle a_1, b_1 \rangle$, $v = \langle a_2, b_2 \rangle$

$u + v = \langle a_1 + a_2, b_1 + b_2 \rangle$

$u - v = \langle a_1 - a_2, b_1 - b_2 \rangle$

$cu = c\langle a_1, b_1 \rangle = \langle ca_1, cb_1 \rangle$

Dot Product $\longrightarrow$ gives only a number

$u \cdot v = a_1 a_2 + b_1 b_2$

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$u = \langle 3, 5 \rangle$ $v = \langle -2, 6 \rangle$

$u + v = \langle 3 + (-2), 5 + 6 \rangle = \langle 1, 11 \rangle$

$u - v = \langle 3 - (-2), 5 - 6 \rangle = \langle 5, -1 \rangle$

$2u = 2\langle 3, 5 \rangle = \langle 6, 10 \rangle$

$-3v = -3\langle -2, 6 \rangle = \langle 6, -18 \rangle$

$u \cdot v = 3 \cdot (-2) + 5 \cdot 6 = -6 + 30 = \boxed{24}$

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$u = \langle -4, 6 \rangle$, $v = \langle 4, -6 \rangle$

1) $u + v = \langle -4+4 , 6+(-6) \rangle = \langle 0, 0 \rangle \Rightarrow$ Zero Vector

2) $u - v = \langle -4-4 , 6-(-6) \rangle = \langle -8, 12 \rangle$

3) $-\frac{1}{2} u = -\frac{1}{2} \langle -4, 6 \rangle = \langle 2, -3 \rangle$

4) $u \cdot v = -4 \cdot 4 + 6 \cdot (-6) = -16 - 36 = \boxed{-52}$

dot

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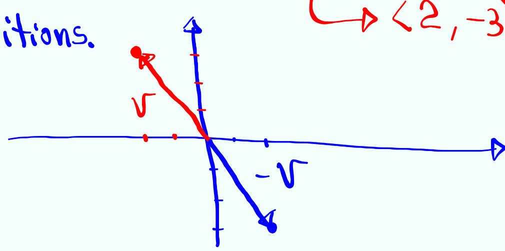
Draw $v = \langle -2, 3 \rangle$ and $-v$ in standard positions.

$-v \to \langle 2, -3 \rangle$



Given $v = \langle 4, 3 \rangle$

1) $|v| = \sqrt{4^2 + 3^2}$
$= \sqrt{25}$
$= \boxed{5}$

2) $v \cdot v = 4 \cdot 4 + 3 \cdot 3$
$= 4^2 + 3^2$
$= 16 + 9$
$= \boxed{25}$

$$\boxed{v \cdot v = |v|^2}$$

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$$\overbrace{\text{magnitude } 1}$$

Unit Vectors have length 1

Show $\langle \frac{3}{5}, \frac{4}{5} \rangle$ is a unit Vector.

$$\left| \langle \tfrac{3}{5}, \tfrac{4}{5} \rangle \right| = \sqrt{\left(\tfrac{3}{5}\right)^2 + \left(\tfrac{4}{5}\right)^2} = \sqrt{\tfrac{9}{25} + \tfrac{16}{25}} = \sqrt{\tfrac{25}{25}} = \boxed{1}$$

Show $\langle \frac{-12}{13}, \frac{5}{13} \rangle$ is a unit Vector.

$$\left| \langle \tfrac{-12}{13}, \tfrac{5}{13} \rangle \right| = \sqrt{\left(\tfrac{-12}{13}\right)^2 + \left(\tfrac{5}{13}\right)^2} = \sqrt{\tfrac{144}{169} + \tfrac{25}{169}} = \sqrt{\tfrac{169}{169}} = \boxed{1}$$

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Special unit Vectors

$$i = \langle 1, 0 \rangle$$

$$j = \langle 0, 1 \rangle$$

$$\langle 5, 3 \rangle = \langle 5, 0 \rangle + \langle 0, 3 \rangle$$

$$= 5\langle 1, 0 \rangle + 3\langle 0, 1 \rangle = 5i + 3j$$

$$\langle -4, 2 \rangle = -4i + 2j$$

$$\langle 6, -3 \rangle = 6i - 3j$$

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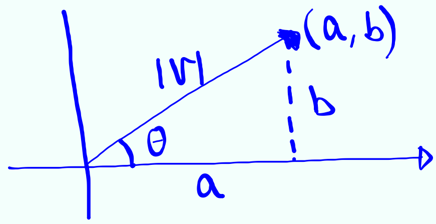
Consider the Vector below



$\sin\theta = \dfrac{b}{|v|}$

$\boxed{b = |v| \sin\theta}$

$\cos\theta = \dfrac{a}{|v|} \Rightarrow \boxed{a = |v|\cos\theta}$

$v = \langle a, b \rangle$

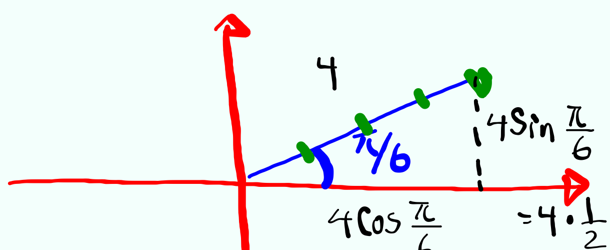
$= a\,i + b\,j$

$$\boxed{v = |v|\cos\theta\, i + |v|\sin\theta\, j}$$

Nov 18-11:13 AM

Vector v has magnitude 4 with direction angle of $\frac{\pi}{6} = 30^\circ$

Draw it



$4\cos\frac{\pi}{6} = 4\cdot\frac{\sqrt{3}}{2} = 2\sqrt{3}$

$= 4\cdot\frac{1}{2} = 2$

find it

$v = \langle 2\sqrt{3}, 2 \rangle$

$= 2\sqrt{3}\, i + 2\, j$

$= 4\cos\frac{\pi}{6}\, i + 4\sin\frac{\pi}{6}\, j$

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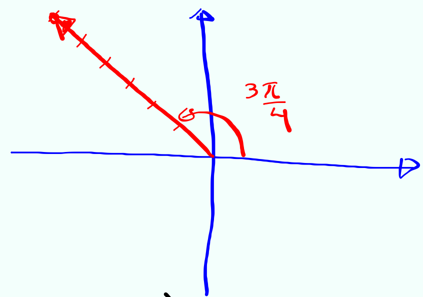
Vector v has a magnitude of 6 with direction angle $\frac{3\pi}{4}$.

1) Draw v



2) Find v

$$v = \langle |v| \cos\theta \,,\, |v| \sin\theta \rangle$$
$$= \langle 6 \cdot \cos\tfrac{3\pi}{4} \,,\, 6 \cdot \sin\tfrac{3\pi}{4} \rangle$$
$$= \langle 6 \cdot -\tfrac{\sqrt{2}}{2} \,,\, 6 \cdot \tfrac{\sqrt{2}}{2} \rangle$$
$$= \langle -3\sqrt{2}, 3\sqrt{2} \rangle = -3\sqrt{2}\,i + 3\sqrt{2}\,j$$

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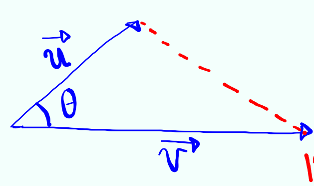
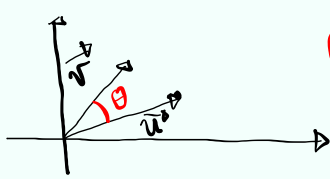
angle between two Vectors



$$\cos\theta = \frac{u \cdot v}{|u|\,|v|}$$

$|u| = \sqrt{29}$ $\quad |v| = \sqrt{18}$

Find the angle between $u = \langle 5,2 \rangle$ and $v = \langle 3,3 \rangle$



$$\cos\theta = \frac{u \cdot v}{|u|\,|v|}$$

$$\cos\theta = \frac{5 \cdot 3 + 2 \cdot 3}{\sqrt{29}\,\sqrt{18}} = \frac{21}{}$$

$$\cos\theta = \frac{21}{\sqrt{522}}$$

$$\theta = \cos^{-1}\left(\frac{21}{\sqrt{522}}\right)$$

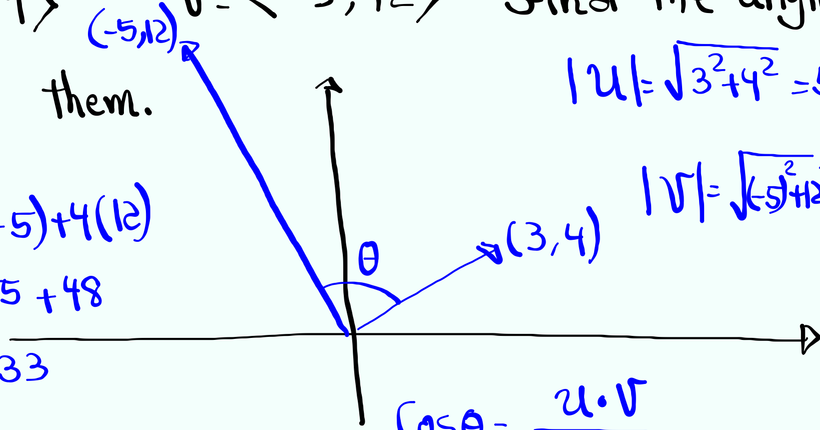
$$\boxed{\theta \approx 23^\circ}$$

Nov 18-11:28 AM

$u = \langle 3, 4 \rangle$ $\quad v = \langle -5, 12 \rangle$ Find the angle between them.

$|u| = \sqrt{3^2 + 4^2} = 5$

$|v| = \sqrt{(-5)^2 + 12^2} = 13$



$u \cdot v = 3(-5) + 4(12)$
$= -15 + 48$
$= 33$

$\cos\theta = \dfrac{u \cdot v}{|u||v|}$
$= \dfrac{33}{5 \cdot 13} = \dfrac{33}{65}$

$\theta = \cos^{-1}\left(\dfrac{33}{65}\right) \approx \boxed{60^\circ}$

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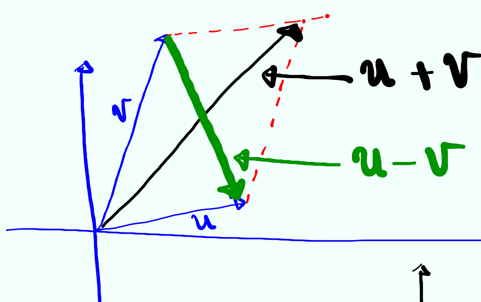

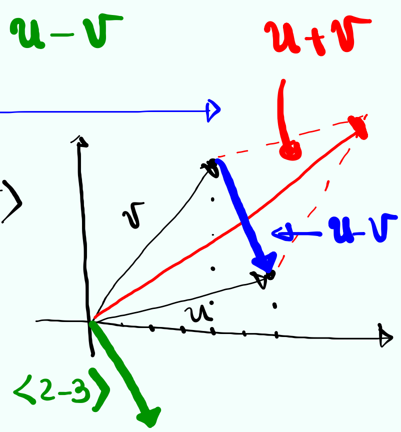

$u = \langle 6, 2 \rangle$ $\quad v = \langle 4, 5 \rangle$

$u + v = \langle 10, 7 \rangle$

$u - v = \langle 2, -3 \rangle$

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